arXiv:nucl-th/9601017v1 12 Jan 1996

Shell Model Study of the Double Beta Decays of ^{76}Ge, ^{82}Se and ^{136}Xe.

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(February 9, 2008)

The lifetimes for the double beta decays of ^{76}Ge, ^{82}Se and ^{136}Xe are calculated using very large shell model spaces. The two neutrino matrix elements obtained are in good agreement with the present experimental data. For $< m_\nu >< 1$ eV we predict the following upper bounds to the half-lives for the neutrinoless mode: $T^{(0\nu)}_{1/2}(Ge) > 1.85\,10^{25} yr.$, $T^{(0\nu)}_{1/2}(Se) > 2.36\,10^{24} yr.$ and $T^{(0\nu)}_{1/2}(Xe) > 1.21\,10^{25} yr.$ These results are the first from a new generation of Shell Model calculations reaching O(10^8) dimensions.

21.60.Cs,23.40.Bw

The importance of the nuclear double beta decay ($\beta\beta$) is well established. The two neutrino mode ($\beta\beta_{(2\nu)}$) that has been proven to be very sensitive to the nuclear correlations, provides a severe test of the nuclear wave functions. The neutrinoless mode ($\beta\beta_{(0\nu)}$) is one of the best probes for the physics beyond the Standard Model. It is particularly suitable to explore the intrinsic properties of the neutrino like its mass and the existence of right-handed weak currents [1–4].

Reasonable values of these lepton number violating parameters can be extracted from the experiment provided that accurate nuclear matrix elements are used. Previous to this work, large Shell Model (SM) calculations of the $\beta\beta$ decay were only possible in ^{48}Ca [1,5,6]. Then, several approximations had to be used in order to study heavier nuclei.

A weak coupling approximation was used by Haxton, Stephenson and Strottman in order to calculate the decays of ^{76}Ge, ^{82}Se and ^{128,130}Te [1]. They also invoked the closure approximation to circumvent the calculation of the 1^+ states in the intermediate nuclei. Therefore, it is very difficult to estimate the uncertainty in their results because there is not an unique method to choose the energy denominator of the two neutrino matrix element. Using an statistical method to determine the energy denominators and, setting $g_A = 1$, they gave reasonable values for the lifetimes in all the cases, except for the Tellurium isotopes.

The quasiparticle random-phase approximation (QRPA) has also been used to study the decay of medium and heavy nuclei [7]. It was found [8,9] that the matrix elements of the Gamow-Teller and double Gamow-Teller operators were very sensitive to the particle-particle interaction. The strength of this interaction (g_{pp}) is treated as a parameter and fitted to the available $\beta_\pm$ data. The introduction of g_{pp} as a phenomenological factor makes it possible to reproduce the observed half-lives. However, the predictivity of the QRPA approach is severely limited because of the large variation of the relevant matrix element $M^{(2\nu)}_{GT}$ in the physical window for g_{pp}. The neutrinoless mode is less sensitive to the correlations, but there still exists a factor of three among several predictions for $M^{(0\nu)}_{GT}$ [10–12]

Very recently the two neutrino mode has been studied in the Shell Model Monte Carlo method (SMMC) [13]. For the decay $^{48}Ca \rightarrow ^{48}Ti$ the matrix element is 0.15 ± 0.07 MeV^{-1}. Using the same interaction, the exact diagonalization gives 0.08 MeV^{-1}. For ^{76}Ge a matrix element 0.13 ± 0.05 MeV^{-1} is found.

The aim of this paper is the study of the $\beta\beta$ decay of ^{76}Ge, ^{82}Se and ^{136}Xe in the SM framework. This study presents a new generation of SM calculations that have recently become feasible due to the availability of a very performing SM code [14] that makes it possible to carry the calculations in an ordinary workstation. Similar importance has the development of a minimal phenomenological treatment of the realistic interactions in order to cure their bad saturation properties [15]. We believe that these calculations represent a real improvement over the old SM and QRPA calculations

For ^{76}Ge and ^{82}Se the Schrodinger equation is solved in a valence space that consists of the following orbitals: $p_{3/2}$, $f_{5/2}$, $p_{1/2}$ and $g_{9/2}$. ^{136}Xe is studied in a valence space that is made of the $d_{5/2}$, $s_{1/2}$, $g_{7/2}$,$d_{3/2}$ and $h_{11/2}$ shells. In table I we show the number of active particles as well as the dimensions reached in each case. In the first space t measures the number of particles that are allowed to jump from the pf subshells to the $g_{9/2}$ orbital. In the case of A=136 it represents the number of particles that can be excited from the lower $g_{7/2}$ and $d_{5/2}$ subshells to the remaining three orbitals. In both cases "full" indicates that no limitations are imposed. Later on we shall analyse the

convergence of the relevant matrix elements from t=0 to the full calculation, except in A=76 where we are limited to t=4(Ge and Se) and t=5(As) because the full calculation is out - but not far - of reach.

We use as starting point a G-matrix from Kuo [16] to calculate the wave functions of the A=76,82 nuclei . To fix the interaction the monopole parameters are fitted to the energy levels of the Ni isotopes and the N=50 isotones [17]. The main monopole changes amount to weaken the interaction among the pf orbits and the $g_{9/2}$ shell. For ^{136}Xe, the interaction is the G-matrix obtained from the the Bonn potential [18] with monopole modifications in order to reproduce the spectroscopy of the N=82 isotones [17]. The secular problem is solved using the Lanczos algorithm.

The eigenstates are used for calculating the matrix elements of the weak interaction inducing the $\beta\beta$ decays. We describe the weak processes in the nucleus by an effective hamiltonian proposed by Doi *et al.* [2], that consists of V and A currents and that is compatible with $SU(2)_L \otimes SU(2)_R \otimes U(1)$ grand unification models.

$$H_w = \frac{G}{\sqrt{2}}[j_L^\mu(J_{L\mu} + \chi J_{R\mu})^+ + j_R^\mu(\eta J_{L\mu} + \lambda J_{R\mu})^+] + h.c., \tag{1}$$

In addition to the right-left and right-right coupling constants, the neutrino mass (implicit in the leptonic currents) also breaks the maximal parity violation of the standard theory. This hamiltonian led us to an scenario where there is no maximal parity violation and where the leptonic number is not preserved. The neutrinoless double beta decay is mediated by (virtual) massive Majorana neutrinos. Moreover the description of the two neutrino mode is essentially the same as that of the Standard Model. This is so because the relevant contributions come from the standard left-left coupling. With regard to the hadronic currents, all the terms up to order v/c are included because, as first noticed by Tomoda *et al.* [19], they contribute significantly to the zero neutrino mode.

$(\beta\beta)_{2\nu}$ *results*: The half-life can be approximated as

$$[T_{\frac{1}{2}}^{(2\nu)}(0^+ -> 0^+)]^{-1} = G\left|M_{GT}^{(2\nu)}\right|^2, \tag{2}$$

where G is a Integral kinematical factor and $M_{GT}^{(2\nu)}$ is the usual energy weighted double Gamow-Teller matrix element. We use the bare Gamow-Teller operator in the definition of the matrix element and use $g_A = 1$ in the integral kinematic factor. The exact definition of this matrix element as well as the algorithm that we use to calculate it can be found in reference [5]. By means of this algorithm we can get a reliable approximation to the Gamow-Teller strength in the intermediate nucleus. However, due to the fact that our model spaces do not include all the spin partner orbits we can not exhaust the 3(N-Z) sum rule. In fact, we find: $S_-^{(Ge)} = 17.14$, $S_-^{(Se)} = 21.66$ and $S_-^{(Xe)} = 52.30$ compared to the 3(N-Z) values 36, 42 and 84 respectively. However, as the remaining strength will appear at high energy its influence in the description of the two neutrino mode should be small.

We summarize our results in table II. This is the only mode whose existence has been clearly confirmed [4] and the lifetimes have been measured for several emitters. There are two sets of matrix elements. In the first one the theoretical energies of the 1^+ states are used whilst, in the second the spectrum of these states is globally shifted in order to place the first 1^+ state at its experimental energy. As it can be seen in table II the matrix elements increase a 20% when the experimental energies are used. The agreement with the experimental data is, in both cases, reasonably good. Further refinements of the interaction are possible but, we do not expect major modifications of the matrix elements.

It is worth comment that our closure matrix elements are very different from those of reference [1], 0.68 compared to 2.56 (^{76}Ge) and 0.74 compared to 1.876 (^{82}Se). These discrepancies also led to quite different effective 1^+ centroids.

In the A=76,82 region the matrix element increases very slowly as the valence space is enlarged. Consequently, the half life decreases and there is a factor two between the t=2 and the final predictions. This behavior is different to that found in the decay of ^{136}Xe, where the matrix element is nearly constant.

It is possible to compare the SM result, 0.14 with the SMMC extrapolation, 0.13±0.05. Both numbers are very close. Nevertheless, the comparison of other relevant quantities ($(M_{GT}^{2\nu})_c$, S_+, $\overline{E}$) is not as good. Any further discussion must be delayed because, although the valence spaces are identical, the effective interactions are different.

$(\beta\beta)_{0\nu}$ *results*: In the closure approximation the half-life of the $0^+ -> 0^+$ decay can be written as

$$\begin{aligned}[T_{\frac{1}{2}}^{(0\nu)}(0^+ -> 0^+]^{-1} = \left|M_{GT}^{(0\nu)}\right|^2 \{ & C_{mm}\left(\frac{<m_\nu>}{m_e}\right)^2 + C_{\lambda\lambda} <\lambda>^2 + C_{\eta\eta} <\eta>^2 + \\ & + C_{\lambda m} <\lambda> \frac{<m_\nu>}{m_e} cos\psi_1 + C_{\eta m} <\eta> \frac{<m_\nu>}{m_e} cos\psi_2 + \\ & + C_{\lambda\eta} <\lambda><\eta> cos(\psi_1 - \psi_2)\},\end{aligned} \tag{3}$$

where $< m_\nu >$, $< \lambda >$ and $< \eta >$ are the effective lepton violating parameters, $\psi_{1(2)}$ are the CP phases and the C_{xy} coefficients are linear combinations of the nine matrix elements and nine integral factors. A clear and comprehensive definition of them all can be found in reference [2].

The nine matrix elements shown in table III are calculated in the light neutrino approximation (see Doi [2]) and using $g_A/g_V = 1.25$. Since this mode has not been observed yet, we take a half-life of $10^{25} yr.$, which is very close to the expected experimental limits [20]. This makes it possible to obtain the upper bounds to the three lepton violating parameters $< m_\nu >$, $< \lambda >$ and $< \eta >$. The results are compiled in table IV. For $< m_\nu >< 1eV$ we find the following lower bounds: $T^{(0\nu)}_{1/2}(Ge) > 1.85\,10^{25} yr.$ $T^{(0\nu)}_{1/2}(Se) > 2.36\,10^{24} yr.$ and $T^{(0\nu)}_{1/2}(Xe) > 1.21\,10^{25} yr.$ Although the result for ^{76}Ge has been obtained in a t=4 calculation, we expect that the matrix element and therefore the half-life are close to convergence, as it is the case in ^{82}Se. Being the $(\beta\beta)_{2\nu}$ mode a background for the detection of the neutrinoless mode the ratio $T^{(2\nu)}_{1/2}/T^{(0\nu)}_{1/2}$ is very important. Notice that, among the three studied nuclei, this ratio is minimum for ^{82}Se and, therefore, it can be considered a good candidate for the detection of the zero neutrino mode.

There are discrepancies among different calculations of the $\beta\beta_{(0\nu)}$ mode. Table V compares the $M^{(0\nu)}_{GT}$ values predicted by several authors. We see that our matrix elements are smaller than the others. The only exception are the new results by Pantis *et al.* [24], that include to some extent the neutron-proton pairing interaction. Based on the reasonable agreement between theory and experiment for the spectroscopy of the regions around these $\beta\beta$ emitters and for the $\beta\beta_{(2\nu)}$ mode, we are also quite confident on our predictions for the zero neutrino mode.

In summary, we have presented the first large scale SM calculations of the $\beta\beta$ decays of ^{76}Ge, ^{82}Se and ^{136}Xe. The agreement with the present experimental data, available for the $\beta\beta_{(2\nu)}$ mode, is reasonable and encouraging. For the $\beta\beta_{(0\nu)}$ mode we have obtained upper bounds to the neutrino mass and to the coupling constants assuming a lifetime of $10^{25} yr.$, and the lifetimes predictions for $< m_\nu >< 1eV$.

Acknowledgments. We thank M. Hjorth-Jensen and T.T. Kuo for making their G-matrix available to us. We thank D. Dean, P. Vogel and A. Zuker for their help. This work has been partly supported by the IN2P3 (France)-CICYT(Spain) agreements and by a grant of DGICYT (Spain) PB93-263.

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TABLE I. Dimensions and active nucleons for each nucleus. See text for the meaning of t.

	N_π	N_ν	$Dimensions$	$Truncation$
^{76}Ge	4	16	8176629	$t=4$
^{76}As	5	15	63788368	$t=5$
^{76}Se	6	14	54625321	$t=4$
^{82}Se	6	20	605367	$full$
^{82}Br	7	19	8353667	$full$
^{82}Kr	8	18	70757366	$full$
^{136}Xe	4	32	1504	$full$
^{136}Cs	5	31	212338	$full$
^{136}Ba	6	30	13139846	$full$

TABLE II. Two neutrino matrix elements and half lives. $M_{GT}^{(2\nu)}$ in MeV^{-1} and T$_{1/2}$ in years.

$Decay$	t	$M_{GT}^{(2\nu)}$		$(M_{GT}^{(2\nu)})_{exp}$	$T_{1/2}^{2\nu}$		$(T_{1/2}^{2\nu})_{exp}$	$(M_{GT}^{(2\nu)})_c$	S_-	S_+
		$\Delta E(exp)$	$\Delta E(th)$		$\Delta E(exp)$	$\Delta E(th)$				
	0	0.000	0.000					0.000	16.73	0.000
$^{76}Ge \rightarrow ^{76}Se$	2	0.112	0.088		$5.678\,10^{21}$	$9.197\,10^{21}$		0.465	17.13	0.146
	4	0.180	0.140		$2.198\,10^{21}$	$3.634\,10^{21}$		0.676	17.14	0.258
	$full$			0.22			$1.80\,10^{21}$			
	0	0.000	0.000					0.000	21.66	0.000
$^{82}Se \rightarrow ^{82}Kr$	2	0.128	0.102		$1.312\,10^{20}$	$2.065\,10^{20}$		0.483	21.61	0.121
	4	0.198	0.155		$5.482\,10^{19}$	$8.946\,10^{19}$		0.745	21.56	0.209
	$full$	0.208	0.164	0.14	$4.968\,10^{19}$	$7.991\,10^{19}$	$1.08\,10^{20}$	0.799	21.55	0.226
	0	0.026	0.028		$2.487\,10^{21}$	$2.455\,10^{21}$		0.106	52.75	0.004
$^{136}Xe \rightarrow ^{136}Ba$	2	0.036	0.039		$1.485\,10^{21}$	$1.265\,10^{21}$		0.178	52.37	0.007
	4	0.032	0.035		$1.879\,10^{21}$	$1.571\,10^{21}$		0.146	52.30	0.008
	$full$	0.031	0.034	< 0.06	$2.003\,10^{21}$	$1.665\,10^{21}$	$> 5.60\,10^{20}$	0.143	52.30	0.008

TABLE III. Nuclear matrix elements for the $(\beta\beta)_{(0\nu)}$ mode. $X^{\omega}_{GT} = 2 - X^{'}_{GT}$, $X^{\omega}_{F} = 2X_F - X^{'}_{F}$.

$M.E.$		$t=0$	$t=2$	$t=4$	$Full$
$M^{(0\nu)}_{GT}$	^{76}Ge	0.721	1.294	1.568	
	^{82}Se	0.505	1.340	1.846	1.970
	^{136}Xe	0.484	0.630	0.649	0.651
χ_F	^{76}Ge	-0.068	-0.098	-0.106	
	^{82}Se	-0.101	-0.107	-0.107	-0.108
	^{136}Xe	-0.172	-0.156	-0.157	-0.158
$\chi^{'}_{GT}$	^{76}Ge	1.074	1.107	1.115	
	^{82}Se	1.069	1.114	1.119	1.120
	^{136}Xe	1.103	1.106	1.099	1.097
$\chi_{F'}$	^{76}Ge	-0.060	-0.102	-0.109	
	^{82}Se	-0.104	-0.112	-0.112	-0.112
	^{136}Xe	-0.184	-0.166	-0.167	-0.167
χ_T	^{76}Ge	0.186	0.043	0.017	
	^{82}Se	0.156	0.049	0.031	0.028
	^{136}Xe	0.039	-0.006	-0.031	-0.031
χ_P	^{76}Ge	-1.435	-0.832	-0.544	
	^{82}Se	1.710	0.852	0.574	0.494
	^{136}Xe	0.898	0.411	0.280	0.256
χ_R	^{76}Ge	0.761	0.707	0.684	
	^{82}Se	0.873	0.706	0.683	0.680
	^{136}Xe	0.780	0.872	0.942	0.955

TABLE IV. Shell Model predictions for the upper bounds to the three lepton violating parameters (LPV): the effective mass $< m_\nu >$ and the coupling constants $< \lambda >$ and $< \eta >$ for $T_{1/2} > 10^{25} yr$.

		$t=0$	$t=2$	$t=4$	$Full$
^{76}Ge	$< m_\nu > (eV)$	2.98	1.60	1.32	
	$< \eta > 10^8$	2.78	1.83	1.65	
	$< \lambda > 10^6$	4.31	2.65	2.24	
^{82}Se	$< m_\nu > (eV)$	1.93	0.72	0.52	0.49
	$< \eta > 10^8$	3.03	1.22	0.86	0.79
	$< \lambda > 10^6$	1.93	0.82	0.61	0.57
^{136}Xe	$< m_\nu > (eV)$	1.47	1.14	1.10	1.10
	$< \eta > 10^8$	3.10	1.77	1.53	1.49
	$< \lambda > 10^6$	1.88	1.53	1.50	1.49

TABLE V. SM versus QRPA $M_{GT}^{(0\nu)}$ matrix elements.

	This Work	*Haxton* [1]	*Tomoda* [10]	*Muto* [11]	*Staut* [12]	*Pantis* [24]
^{76}Ge	1.568	4.180	3.355	3.014	10.910	1.846
^{82}Se	1.970	3.450	3.055	2.847		1.153
^{136}Xe	0.651		1.120			1.346